2 Indices and Logarithms

INTRODUCTION

In previous classes, we have read that if a is any real number and n is a natural number, then

$$a^n = a \times a \times a \ldots. \; n \text{ times}$$

where a is called the **base**, n is called the **exponent** or **index** and a^n is the **exponential form.** We defined

$$a^0 = 1 \text{ and } a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}, a \neq 0.$$ We note that $0°$ is not defined. It is called an indeterminant form.

For example:

(*i*) $3^0 = 1,\ 3^5 = 3.3.3.3.3 = 243,\ 3^{-5} = \frac{1}{3^5} = \frac{1}{243}$.

(*ii*) $\left(\frac{2}{5}\right)^0 = 1, \left(\frac{2}{5}\right)^3 = \frac{2}{5}.\frac{2}{5}.\frac{2}{5} = \frac{8}{125}, \left(\frac{2}{5}\right)^{-3} = \frac{1}{\left(\frac{2}{5}\right)^3} = \frac{1}{\frac{8}{125}} = \frac{125}{8}$.

(*iii*) $(-2)^0 = 1, (-2)^3 = (-2)\,(-2)\,(-2) = -8, (-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$.

Also, we have read the laws of exponents.

If a, b are rational numbers and m, n are integers, then the following results hold:

(*i*) $a^m \,.\, a^n = a^{m+n}$ (*ii*) $(a^m)^n = a^{mn}$ (*iii*) $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

(*iv*) $a^m \,.\, b^m = (ab)^m$ (*v*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ (*vi*) $a^0 = 1, a \neq 0$

(*vii*) $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}, a \neq 0$ (*viii*) $a^n = b^n, n \neq 0 \Rightarrow a = b$

(*ix*) $a^m = a^n, a \neq 1 \Rightarrow m = n.$

In this chapter, we shall extend these laws when the base is a positive real number and the exponents are rational numbers.

FRACTIONAL INDICES (OR SURDS)

We know that the square root of a positive real number a is that number which when multiplied by itself gives a as the product.

Thus, if b is the square root a, then $b \times b = a$ *i.e.* $b^2 = a$.

The square root of the number a is denoted by $\sqrt{a}$, so $b = \sqrt{a}$.

The concept of square root can be extended to cube root, fourth root, ..., nth root, where n is a natural number.

Let a be a positive real number and n be a natural number, then $\sqrt[n]{a} = b$ if and only if $b^n = a$, $b > 0$.

For example: $\sqrt[3]{8} = 2$ because $2^3 = 8$; $\sqrt[5]{243} = 3$ because $3^5 = 243$. Note that the symbol '$\sqrt{\ }$' used in $\sqrt{5}, \sqrt[3]{8}, \sqrt[5]{243}$ is called the **radical sign.**

In the language of exponents, we write $\sqrt[n]{a} = a^{\frac{1}{n}}$.

So, in particular, we write $\sqrt[3]{8} = 8^{\frac{1}{3}}, \sqrt[5]{243} = (243)^{\frac{1}{5}}$ etc.

Now let us try to understand what is $8^{\frac{2}{3}}$?

There are two ways:

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4; \qquad 8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = (64)^{\frac{1}{3}} = \sqrt[3]{64} = 4.$$

This leads us to the following definition:

If $a > 0$ is a real number and m, n are integers, $n > 0$, m, n have no common factors except 1, then

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}.$$

LAWS OF EXPONENTS FOR REAL NUMBERS

Laws of exponents for real numbers are:

If a, b are positive real numbers and m, n are rational numbers, then the following results hold:

(i) $a^m . a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iv) $a^m . b^m = (ab)^m$ (v) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$ (vi) $a^{-n} = \left(\dfrac{1}{a}\right)^n = \dfrac{1}{a^n}$

(vii) $a^n = b^n, n \neq 0 \Rightarrow a = b$ (viii) $a^m = a^n \Rightarrow m = n$ provided $a \neq 1$.

Note

If p and q are different positive prime integers, then $p^m q^n = p^l q^k \Rightarrow m = l$ and $n = k$.

ILLUSTRATIVE EXAMPLES

Example 1. *Simplify the following:*

(i) $(3x^4y^3)(18x^3y^5)$ (ii) $\dfrac{3x^4y^3}{18x^3y^5}$ (iii) $\left(-\dfrac{2x^2}{y^3}\right)^3$ (iv) $\sqrt[3]{27^{-2}}$.

Solution. (i) $(3x^4y^3)(18x^3y^5) = 3.18.x^4.x^3.y^3.y^5 = 54x^7y^8$.

(ii) $\dfrac{3x^4y^3}{18x^3y^5} = \dfrac{3}{18}\cdot\dfrac{x^4}{x^3}\cdot\dfrac{y^3}{y^5} = \dfrac{1}{6}.x^{4-3}.\dfrac{1}{y^{5-3}} = \dfrac{x}{6y^2}$.

(iii) $\left(-\dfrac{2x^2}{y^3}\right)^3 = (-2)^3.\dfrac{(x^2)^3}{(y^3)^3} = -8.\dfrac{x^6}{y^9}$.

(iv) $\sqrt[3]{27^{-2}} = (27^{-2})^{1/3} = (27)^{-2/3} = (3^3)^{-2/3} = 3^{3 \times (-2/3)} = 3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$.

Example 2. *Simplify the following:*

(*i*) $\left[\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2$ (*ii*) $(256)^{-\left(4^{-\frac{3}{2}}\right)}$ (*iii*) $\sqrt[4]{28} \div \sqrt[3]{7}$.

Solution. (*i*) $\left[\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2 = \left[(625)^{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{4}\right)}\right]^2 = \left[(625)^{\frac{1}{8}}\right]^2$

$$= (625)^{\frac{1}{8}\times 2} = (625)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{4\times\frac{1}{4}} = 5^1 = 5.$$

(*ii*) Note that $4^{-\frac{3}{2}} = (2^2)^{-\frac{3}{2}} = 2^{2\times\left(-\frac{3}{2}\right)} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

$$\therefore\ (256)^{-\left(4^{-\frac{3}{2}}\right)} = (256)^{-\frac{1}{8}} = (2^8)^{-\frac{1}{8}} = 2^{8\times\left(-\frac{1}{8}\right)} = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}.$$

(*iii*) $\sqrt[4]{28} \div \sqrt[3]{7} = (28)^{\frac{1}{4}} \div (7)^{\frac{1}{3}}$

L.C.M. of 4 and 3 = 12, so write each number with exponent 12.

$$(28)^{\frac{1}{4}} = ((28)^3)^{\frac{1}{12}} \text{ and } 7^{\frac{1}{3}} = (7^4)^{\frac{1}{12}}$$

$$\therefore\ \sqrt[4]{28} \div \sqrt[3]{7} = \frac{((28)^3)^{\frac{1}{12}}}{(7^4)^{\frac{1}{12}}} = \left(\frac{(28)^3}{7^4}\right)^{\frac{1}{12}} = \left(\frac{28\times 28\times 28}{7\times 7\times 7\times 7}\right)^{\frac{1}{12}}$$

$$= \left(\frac{4\times 4\times 4}{7}\right)^{\frac{1}{12}} = \left(\frac{64}{7}\right)^{\frac{1}{12}} = \sqrt[12]{\frac{64}{7}}.$$

Example 3. *Simplify:* $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$.

Solution. $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

$$= \left(\left(\frac{3}{2}\right)^4\right)^{-\frac{3}{4}} \times \left[\left(\left(\frac{5}{3}\right)^2\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{4\times\left(-\frac{3}{4}\right)} \times \left[\left(\frac{5}{3}\right)^{2\times\left(-\frac{3}{2}\right)} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left[\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{2}{3}\right)^3 \times \left[\left(\frac{3}{5}\right)^3 \div \left(\frac{2}{5}\right)^3\right] = \frac{2^3}{3^3} \times \left[\frac{3^3}{5^3} \div \frac{2^3}{5^3}\right] \qquad \left[\because a^{-n} = \left(\frac{1}{a}\right)^n\right]$$

$$= \frac{2^3}{3^3} \times \left(\frac{3^3}{5^3} \times \frac{5^3}{2^3}\right) = \frac{2^3}{3^3} \times \frac{3^3}{2^3} = 1.$$

Example 4. *Find the value of* $\dfrac{4}{(216)^{-\frac{2}{3}}}+\dfrac{1}{(256)^{-\frac{3}{4}}}+\dfrac{2}{(243)^{-\frac{1}{5}}}$.

Solution. $\dfrac{4}{(216)^{-\frac{2}{3}}}+\dfrac{1}{(256)^{-\frac{3}{4}}}+\dfrac{2}{(243)^{-\frac{1}{5}}}$

$$= 4(216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + 2(243)^{\frac{1}{5}}$$

$$= 4(6^3)^{\frac{2}{3}} + (4^4)^{\frac{3}{4}} + 2(3^5)^{\frac{1}{5}} = 4\left(6^{3\times\frac{2}{3}}\right) + 4^{4\times\frac{3}{4}} + 2\left(3^{5\times\frac{1}{5}}\right)$$

$$= 4(6^2) + 4^3 + 2(3^1) = 4 \times 36 + 64 + 6$$

$$= 144 + 64 + 6 = 214.$$

Example 5. *Evaluate:* $\sqrt{\frac{1}{4}} + (0.01)^{-1/2} - (27)^{2/3}$. *Leave your answer as a fraction.*

Solution. $\sqrt{\frac{1}{4}} + (0.01)^{-1/2} - (27)^{2/3} = \frac{1}{2} + \left(\frac{1}{100}\right)^{-1/2} - (3^3)^{2/3}$

$$= \frac{1}{2} + (100)^{1/2} - 3^2 \qquad \left[\because a^{-n} = \left(\frac{1}{a}\right)^n\right]$$

$$= \frac{1}{2} + ((10)^2)^{1/2} - 9$$

$$= \frac{1}{2} + 10 - 9 = \frac{1}{2} + 1 = 1\frac{1}{2}.$$

Example 6. *Simplify:* $\left(\frac{1}{4}\right)^{-2} - 3(8)^{2/3}(4)^0 + \left(\frac{9}{16}\right)^{-1/2}$.

Solution. $\left(\frac{1}{4}\right)^{-2} - 3(8)^{2/3}(4)^0 + \left(\frac{9}{16}\right)^{-1/2} = (4)^2 - 3.(2^3)^{2/3}.1 + \left(\frac{16}{9}\right)^{1/2}$

$$\left[\because a^{-n} = \left(\frac{1}{a}\right)^n \text{ and } a^0 = 1\right]$$

$$= 16 - 3 \times 2^2 \times 1 + \left(\left(\frac{4}{3}\right)^2\right)^{1/2}$$

$$= 16 - 3 \times 4 + \frac{4}{3} = 16 - 12 + \frac{4}{3}$$

$$= 4 + \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}.$$

Example 7. *Simplify the following:*

(*i*) $\dfrac{5^{n+2} - 6.5^{n+1}}{13.5^n - 2.5^{n+1}}$ (*ii*) $\left(\dfrac{x^m}{x^n}\right)^{m+n}\left(\dfrac{x^n}{x^l}\right)^{n+l}\left(\dfrac{x^l}{x^m}\right)^{l+m}$

(*iii*) $\left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right)\left(x^{\frac{2}{3}} - 1 + x^{-\frac{2}{3}}\right)$.

Solution. (*i*) $\dfrac{5^{n+2} - 6.5^{n+1}}{13.5^n - 2.5^{n+1}} = \dfrac{5^n.5^2 - 6.5^n.5^1}{13.5^n - 2.5^1.5^n}$

$$= \frac{5^n(5^2 - 6.5)}{5^n(13 - 2.5)} = \frac{25 - 30}{13 - 10} = \frac{-5}{3} = -\frac{5}{3}.$$

(ii) $\left(\frac{x^m}{x^n}\right)^{m+n}\left(\frac{x^n}{x^l}\right)^{n+l}\left(\frac{x^l}{x^m}\right)^{l+m}$

$= (x^{m-n})^{m+n}.(x^{n-l})^{n+l}.(x^{l-m})^{l+m}$

$= x^{m^2-n^2}.x^{n^2-l^2}.x^{l^2-m^2}$

$= x^{m^2-n^2+n^2-l^2+l^2-m^2} = x^0 = 1.$

(iii) $\left(x^{\frac{1}{3}}+x^{-\frac{1}{3}}\right)\left(x^{\frac{2}{3}}-1+x^{-\frac{2}{3}}\right) = x^{\frac{1}{3}}.x^{\frac{2}{3}} - x^{\frac{1}{3}} + x^{\frac{1}{3}}.x^{-\frac{2}{3}} + x^{-\frac{1}{3}}.x^{\frac{2}{3}} - x^{-\frac{1}{3}} + x^{-\frac{1}{3}}.x^{-\frac{2}{3}}$

$= x^1 - x^{\frac{1}{3}} + x^{-\frac{1}{3}} + x^{\frac{1}{3}} - x^{-\frac{1}{3}} + x^{-1}$

$= x + x^{-1} = x + \frac{1}{x}.$

Example 8. *If $a = b^{2x}$, $b = c^{2y}$ and $c = a^{2z}$, prove that $xyz = \frac{1}{8}$.*

Solution. Given $a = b^{2x}$ ...(i) $\quad b = c^{2y}$ ...(ii) $\quad c = a^{2z}$ ...(iii)

Substituting the value of b from (ii) in (i), we get

$a = (c^{2y})^{2x} = c^{4xy}$ ...(iv)

Substituting the value of c from (iii) in (iv), we get

$a = (a^{2z})^{4xy} = a^{8xyz}$

$\Rightarrow \quad a^1 = a^{8xyz} \Rightarrow 1 = 8xyz$ (Assume $a > 0, a \neq 1$)

$\Rightarrow \quad xyz = \frac{1}{8}.$

Example 9. *If $a^x = b^y = c^z$ and $b^2 = ac$, prove that $y = \frac{2xz}{z+x}$.*

Solution. Let $a^x = b^y = c^z = k$ (say), then

$a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}$ and $c = k^{\frac{1}{z}}.$

Given $\quad b^2 = ac \Rightarrow \left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}}.k^{\frac{1}{z}}$

$\Rightarrow \quad k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}} \Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\Rightarrow \quad \frac{2}{y} = \frac{z+x}{xz} \Rightarrow y = \frac{2xz}{z+x}.$

Example 10. *If $abc = 1$, show that*

$$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1.$$

Solution. $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$

$= \frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}}$

$= \frac{b}{b+ab+1} + \frac{1}{1+b+ab} + \frac{1}{1+\frac{1}{ab}+\frac{1}{a}}$ $\quad \left(\because abc = 1 \Rightarrow \frac{1}{c} = ab \text{ and } c = \frac{1}{ab}\right)$

$= \frac{b}{1+b+ab} + \frac{1}{1+b+ab} + \frac{ab}{ab+1+b} = \frac{b+1+ab}{1+b+ab} = 1.$

Example 11. *If* $x = \sqrt[3]{28}$ *and* $y = \sqrt[3]{27}$ *, find the value of* $x + y - \dfrac{1}{x^2 + xy + y^2}$.

Solution. $x + y - \dfrac{1}{x^2 + xy + y^2} = x + y - \dfrac{x - y}{(x - y)(x^2 + xy + y^2)}$ (Note this step)

$$= x + y - \frac{x - y}{x^3 - y^3}$$

$$= x + y - \frac{x - y}{((28)^{1/3})^3 - ((27)^{1/3})^3}$$

$$= x + y - \frac{x - y}{28 - 27} = x + y - \frac{x - y}{1}$$

$$= x + y - (x - y) = 2y$$

$$= 2 \times \sqrt[3]{27} = 2 \times (27)^{1/3}$$

$$= 2 \times (3^3)^{\frac{1}{3}} = 2 \times 3^1 = 6.$$

Example 12. *Given* $1176 = 2^p \,.\, 3^q \,.\, 7^r$*, find*

(*i*) *the numerical values of* p, q *and* r (*ii*) *the value of* $2^p.3^q.7^{-r}$ *as a fraction.*

Solution. (*i*) Given $1176 = 2^p.3^q.7^r$

$\Rightarrow$ $2 \times 2 \times 2 \times 3 \times 7 \times 7 = 2^p.3^q.7^r$

$\Rightarrow$ $2^3.3^1.7^2 = 2^p.3^q.7^r$

$\Rightarrow$ $p = 3, q = 1$ and $r = 2$. (See note page 11)

(*ii*) $2^p.3^q.7^{-r} = 2^3.3^1.7^{-2} = \dfrac{8 \times 3}{7^2} = \dfrac{24}{49}$.

Example 13. *Solve the following equations for x:*

(*i*) $4^{2x} = \dfrac{1}{32}$ (*ii*) $\sqrt{\left(\dfrac{3}{5}\right)^{1-2x}} = 4\dfrac{17}{27}$.

Solution. (*i*) Given $4^{2x} = \dfrac{1}{32} \Rightarrow (2^2)^{2x} = \left(\dfrac{1}{2}\right)^5$

$\Rightarrow$ $2^{4x} = 2^{-5}$ $\left[\because \left(\dfrac{1}{a}\right)^n = a^{-n}\right]$

$\Rightarrow$ $4x = -5 \Rightarrow x = -\dfrac{5}{4}$.

(*ii*) Given $\sqrt{\left(\dfrac{3}{5}\right)^{1-2x}} = 4\dfrac{17}{27} \Rightarrow \left(\left(\dfrac{3}{5}\right)^{1-2x}\right)^{1/2} = \dfrac{125}{27}$

$\Rightarrow$ $\left(\dfrac{3}{5}\right)^{\frac{1-2x}{2}} = \left(\dfrac{5}{3}\right)^3 \Rightarrow \left(\dfrac{3}{5}\right)^{\frac{1-2x}{2}} = \left(\dfrac{3}{5}\right)^{-3}$ $\left[\because \left(\dfrac{1}{a}\right)^n = a^{-n}\right]$

$\Rightarrow$ $\dfrac{1-2x}{2} = -3 \Rightarrow 1 - 2x = -6$

$\Rightarrow$ $-2x = -6 - 1 \Rightarrow -2x = -7 \Rightarrow x = \dfrac{7}{2}$.

EXERCISE 2.1

Simplify the following (1 to 9):

1. (*i*) $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}}$ (*ii*) $\left(1\dfrac{61}{64}\right)^{-\frac{2}{3}}$.

2. (*i*) $\dfrac{3a}{b^{-1}} + \dfrac{2b}{a^{-1}}$ (*ii*) $5^0 \times 4^{-1} + 8^{1/3}$.

3. (i) $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$ (ii) $(0.027)^{-\frac{1}{3}}$.

4. (i) $\left[8^{-\frac{4}{3}} \div 2^{-2}\right]^{1/2}$ (ii) $\left(\frac{27}{8}\right)^{2/3} - \left(\frac{1}{4}\right)^{-2} + 5^0$.

5. (i) $(3^2)^0 + 3^{-4} \times 3^6 + \left(\frac{1}{3}\right)^{-2}$ (ii) $9^{5/2} - 3.(5)^0 - \left(\frac{1}{81}\right)^{-1/2}$.

6. (i) $16^{3/4} + 2\left(\frac{1}{2}\right)^{-1} (3)^0$ (ii) $(81)^{3/4} - \left(\frac{1}{32}\right)^{-2/5} + (8)^{1/3}\left(\frac{1}{2}\right)^{-1} (2)^0$.

7. (i) $\frac{\sqrt{2^2} \times \sqrt[4]{256}}{\sqrt[3]{64}} - \left(\frac{1}{2}\right)^{-2}$ (ii) $\frac{3^{-\frac{6}{7}} \times 4^{-\frac{3}{7}} \times 9^{\frac{3}{7}} \times 2^{\frac{6}{7}}}{2^2 + 2^0 + 2^{-2}}$.

8. (i) $\frac{(32)^{\frac{2}{5}} \times (4)^{-\frac{1}{2}} \times (8)^{\frac{1}{3}}}{2^{-2} \div (64)^{-1/3}}$ (ii) $\frac{5^{2(x+6)} \times (25)^{-7+2x}}{(125)^{2x}}$.

9. (i) $\left(\frac{x^m}{x^n}\right)^l \cdot \left(\frac{x^n}{x^l}\right)^m \cdot \left(\frac{x^l}{x^m}\right)^n$ (ii) $\left(\frac{x^{a+b}}{x^c}\right)^{a-b} \cdot \left(\frac{x^{b+c}}{x^a}\right)^{b-c} \cdot \left(\frac{x^{c+a}}{x^b}\right)^{c-a}$.

10. Prove the following:

(i) $(a+b)^{-1}\ (a^{-1} + b^{-1}) = \frac{1}{ab}$ (ii) $\frac{x+y+z}{x^{-1}y^{-1} + y^{-1}z^{-1} + z^{-1}x^{-1}} = xyz.$

11. If $a = c^z$, $b = a^x$ and $c = b^y$, prove that $xyz = 1$.

12. If $a = xy^{p-1}$, $b = xy^{q-1}$ and $c = xy^{r-1}$, prove that
$a^{q-r}.b^{r-p}.c^{p-q} = 1$.

13. If $2^x = 3^y = 6^{-z}$, prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$.

14. If $2^x = 3^y = 12^z$, prove that $x = \frac{2yz}{y-z}$.

15. Evaluate $x^{1/2}.y^{-1}.z^{2/3}$ when $x = 9$, $y = 2$ and $z = 8$.

16. If $x^4y^2z^3 = 49392$, find the values of x, y and z, where x, y and z are different positive primes.

17. Solve the following equations for x:

(i) $5^{2x+3} = 1$ (ii) $(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$

(iii) $\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$ (iv) $(\sqrt[3]{4})^{2x+\frac{1}{2}} = \frac{1}{32}$.

18. If $\frac{9^n.3^2.3^n - (27)^n}{3^{3m}.2^3} = \frac{1}{27}$, prove that $m = 1 + n$.

Answers

1. (i) $\frac{8}{27}$ (ii) $\frac{16}{25}$ 2. (i) $5ab$ (ii) $2\frac{1}{4}$

3. (i) $2\frac{1}{2}$ (ii) $3\frac{1}{3}$ 4. (i) $\frac{1}{2}$ (ii) $-12\frac{3}{4}$

5. (i) 19 (ii) 231 6. (i) 12 (ii) 27

7. (*i*) -2 (*ii*) $\frac{4}{21}$

8. (*i*) 4 (*ii*) $\frac{1}{25}$

9. (*i*) 1 (*ii*) 1

15. 6

16. $x = 2, y = 3, z = 7$

17. (*i*) $-\frac{3}{2}$ (*ii*) 4 (*iii*) -7 (*iv*) -4